

## A. Ballots and Preference Schedules

1. The management of the XYZ Corporation has decided to treat their office staff to dinner. The choice of restaurants is The Atrium (*A*), Blair's Kitchen (*B*), The Country Cookery (*C*), and Dino's Steak House (*D*). Each of the 12 staff members is asked to submit a preference ballot listing his or her first, second, third, and fourth choices among these restaurants. The resulting preference ballots are as follows:

Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot
1st <i>A</i>	1st <i>C</i>	1st <i>B</i>	1st <i>C</i>	1st <i>C</i>	1st <i>C</i>	1st <i>A</i>	1st <i>C</i>	1st <i>A</i>	1st <i>A</i>	1st <i>C</i>	1st <i>A</i>
2nd <i>B</i>	2nd <i>B</i>	2nd <i>D</i>	2nd <i>B</i>	2nd <i>B</i>	2nd <i>B</i>	2nd <i>B</i>	2nd <i>B</i>	2nd <i>B</i>	2nd <i>B</i>	2nd <i>B</i>	2nd <i>B</i>
3rd <i>C</i>	3rd <i>D</i>	3rd <i>C</i>	3rd <i>A</i>	3rd <i>A</i>	3rd <i>D</i>	3rd <i>C</i>	3rd <i>A</i>	3rd <i>C</i>	3rd <i>C</i>	3rd <i>D</i>	3rd <i>C</i>
4th <i>D</i>	4th <i>A</i>	4th <i>A</i>	4th <i>D</i>	4th <i>D</i>	4th <i>A</i>	4th <i>D</i>	4th <i>D</i>	4th <i>D</i>	4th <i>D</i>	4th <i>A</i>	4th <i>D</i>

- (a) How many first-place votes are needed for a majority?  
 (b) Which restaurant has the most first-place votes? Is it a majority or a plurality?  
 (c) Write out the preference schedule for this election.
3. An election is held to choose the Chair of the Mathematics Department at Tasmania State University. The candidates are Professors Argand, Brandt, Chavez, Dietz, and Epstein (*A*, *B*, *C*, *D*, and *E*). The preference schedule for the election is as follows:

Number of voters	5	3	5	3	2	3
1st choice	<i>A</i>	<i>A</i>	<i>C</i>	<i>D</i>	<i>D</i>	<i>B</i>
2nd choice	<i>B</i>	<i>D</i>	<i>E</i>	<i>C</i>	<i>C</i>	<i>E</i>
3rd choice	<i>C</i>	<i>B</i>	<i>D</i>	<i>B</i>	<i>B</i>	<i>A</i>
4th choice	<i>D</i>	<i>C</i>	<i>A</i>	<i>E</i>	<i>A</i>	<i>C</i>
5th choice	<i>E</i>	<i>E</i>	<i>B</i>	<i>A</i>	<i>E</i>	<i>D</i>

- (a) How many people voted in this election?  
 (b) How many first-place votes are needed for a majority?  
 (c) Which candidate had the most first-place votes?  
 (d) Which candidate had the most last-place votes?

6. The student body at Eureka High School is having an election for Homecoming Queen. The candidates are Alicia, Brandy, Cleo, and Dionne (*A*, *B*, *C*, *D*, and *E*). The preference schedule for the election is as follows:

Number of voters	153	102	55	202	108	20	110	160	175	155
1st choice	<i>A</i>	<i>A</i>	<i>A</i>	<i>B</i>	<i>B</i>	<i>B</i>	<i>C</i>	<i>C</i>	<i>D</i>	<i>D</i>
2nd choice	<i>C</i>	<i>B</i>	<i>D</i>	<i>D</i>	<i>C</i>	<i>C</i>	<i>A</i>	<i>B</i>	<i>A</i>	<i>B</i>
3rd choice	<i>B</i>	<i>D</i>	<i>C</i>	<i>A</i>	<i>D</i>	<i>A</i>	<i>D</i>	<i>A</i>	<i>C</i>	<i>C</i>
4th choice	<i>D</i>	<i>C</i>	<i>B</i>	<i>C</i>	<i>A</i>	<i>D</i>	<i>B</i>	<i>D</i>	<i>B</i>	<i>A</i>

Suppose that the election rules are that when there is a candidate with a majority of the first-place votes, she is the winner. Otherwise, all candidates with 25% or less of the first-place votes are eliminated and the ballots are recounted.

- (a) Which candidates are eliminated in this election?  
 (b) Find the preference schedule for the recount.  
 (c) Which candidate is the majority winner after the recount?

## B. Plurality Method

11. This exercise refers to the election for Homecoming Queen at Eureka High School discussed in Exercise 6.
- (a) Find the winner(s) of the election under the plurality method.  
 (b) Suppose that in case of a tie, the winner is decided by choosing the candidate with the fewest last-place votes. In this case, which candidate would win the election?
15. Consider an election with 721 voters.
- (a) What is the smallest number of votes needed to be a majority candidate?  
 (b) If there are 5 candidates, what is the smallest number of votes that a plurality candidate could have?  
 (c) If there are 10 candidates, what is the smallest number of votes that a plurality candidate could have?